Automated Market Maker Liquidity

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April 2021

1 Automated Market Maker (AMM)

An Automated Market Maker (AMM) is a contract that holds two or more assets in a "pool". A pool acts as a decentralised exchange, facilitating exchange between the two assets without the need for an orderbook. For most types of AMMs, the quantities of two assets will always satisfy the equation:

$$A \times B = k \tag{1}$$

where A is the number of token A in the pool, and B is the number of token B. K is held constant.

For example, if we start a pool with 100 of token A, and 1 of token B with an initial price A/B of $1/100 = 0.01$, and $k = 100$. If someone goes to buy 20 of token A from the pool then this would reduce A_n to 80, requiring that B_n increases to $B_n = \frac{100}{80} = 1.25$. This means that those 20 of token A would cost 0.25 of token B. A trade price A/B of $0.25/20 = 0.0125$. You can see where the slippage is coming in, the price of the tokens increases with the number of tokens you are buying.

1.1 Liquidity

We can generalise this further, with an initial pool state:

$$A_1 B_1 = k \tag{2}$$

and initial price $P_1 = B_1/A_1$.

Suppose the number of token A in the pool changes by some factor x such that $A_2 = xA_1$. If someone buys 10% of tokens, $A_2 = 0.9A_1$. As k is constant, the updated pool will be

$$A_1 B_1 = A_2 B_2 = x A_1 B_2 \tag{3}$$

so

$$x A_1 B_2 = A_1 B_1 \tag{4}$$

then

$$B_2 = \frac{B_1}{x} \tag{5}$$

This makes sense as:

$$A_2B_2 = xA_1\frac{B_1}{x} = A_1B_1 \tag{6}$$

The updated price P_2 will be

$$P_2 = B_2/A_2 = \frac{\frac{B_1}{x}}{xA_1} = \frac{1}{x^2}\frac{B_1}{A_1} = \frac{P_1}{x^2} \tag{7}$$

For example, if you buy 20% of the remaining token A in the pool, the price change of token A in the pool will be $\frac{1}{(1-0.2)^2} = 1.5625$, a +56.25% increase. Or if you sell heaps of token A to the pool, increasing its token A balance by 10x, then this is a price change of $\frac{1}{10^2} = 0.01$, a 99% decrease in price.

1.2 Money received from selling Y tokens

If we sell y of token A via the pool, how many of token B will we receive? Let $A_2 = xA_1 = A_1 + y$, because we're adding y tokens to the pool. So $x = 1 + \frac{y}{A_1}$. From (5) we have

$$B_2 = \frac{B_1}{x} = B_1(\frac{1}{1+\frac{y}{A_1}}) \tag{8}$$

So $B_2 = B_1(\frac{1}{1+\frac{y}{A_1}})$. In otherwords, selling $y = A_2 - A_1$ of token A, gives z of token B:

$$z = B_2 - B_1 = B_1(\frac{1}{1+\frac{y}{A_1}} - 1) \tag{9}$$

For example, if there is a pool with 20 of token A and 5 of token B, and we sell 2 of token A then we get

$$z = B_1(\frac{1}{1+\frac{y}{A_1}} - 1) = 5(\frac{1}{1+\frac{2}{20}} - 1) = -0.45454... \tag{10}$$

Checking:

$$20 * 5 = 100 = k \tag{11}$$

$$(20+2) * (5 - 0.4545..) = 100 = k \tag{12}$$